Agriculture and forestry with a long term climate constraint: The case of Norway

*** Ongoing work ***

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The project

"Strategies to reduce greenhouse gas emissions in Norwegian agriculture"

- An alternative use of agricultural land is forestry (sequestration of CO_2). This calls for a dynamic analysis

Develop a simple dynamic model to analyze these issues. More will be added later, and calibrated to Norwegian data.

Which aspects are (a priori) important, which are not?

Basics & Agriculture

Time $t = 0, ..., T$ (discrete, finite horizon). Fixed piece of land of size $\bar{L}$ (extension: distinguish between various productivities).

There are two agricultural products, plants and meat. Plants $Y_v(t)$ are produced with land $L_v(t)$ and fertilizers $N(t)$ as inputs

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Production of meat $Y_m(t)$ requires only plants $Y_{vm}(t)$ as input

$$Y_m(t) = f_m(Y_{vm}(t)) \qquad (2)$$

Agriculture (cont.)

Consumers: An inverse demand (currently linear) by consumers for plant- and meat products:

$$P_v(t) = \alpha_v - \beta_v X_v(t) \text{ and } P_m(t) = \alpha_m - \beta_m X_m(t) \tag{3}$$

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It must hold that

$$Y_{vm}(t) + X_v(t) \leq Y_v(t) \text{ and } X_m(t) \leq Y_m(t). \tag{4}$$

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Decision: $L_p(t)$ land planted with trees. Therefore

$$L_c(t + 1, 0) = L_p(t) \tag{7}$$

Forestry

Land constraint:

$$L_p(t) + \sum_{a=0}^{A} \left(L_c(t,a) - L_f(t,a)\right) + L_v(t) \leq \bar{L} \tag{8}$$

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Area covered with forest of age a has standing volume $w(a)$ per unit area. Harvested area $L_f(t,a)$ therefore yields volume felled

$$V(t) = \sum_{a=0}^{A} \left(w(a) \cdot L_f(t,a)\right). \tag{9}$$

Atmosphere

Let $S(t)$ be what is sequestrated from the atmosphere in forestry in period t.

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Carbon emissions $E_C(t)$ in period t, amount to

$$E_C(t) = E_{CX}(t) - F_C(q_C(t)) - S(t) \tag{11}$$

$E_{CX}(t)$ are business as usual carbon emissions from other Norwegian sectors (global emissions will also enter above),
$F_C(q_C(t))$ are emissions abated when spending $q_C(t)$ dollars on abatement (increasing, concave)

Atmosphere

Emissions of methane CH_4 is given by

$$E_M(t) = E_{MX}(t) - F_M(q_M(t)) + \varepsilon_M Y_{vm}(t)$$

Similarly N_2O emissions are given by

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Cannot abate more than you have

$$\begin{aligned} F_C(q_C(t)) &\leq E_{CX}(t) \\ F_M(q_M(t)) &\leq E_{MX}(t) \\ F_N(q_N(t)) &\leq E_{NX}(t) \end{aligned}$$

Atmosphere

The atmospheric concentrations

$$\begin{aligned} C_C(t+1) &= \alpha_C \cdot C_C(t) + \beta_C \cdot E_C(t) \\ C_M(t+1) &= \alpha_M \cdot C_M(t) + \beta_M \cdot E_M(t) \\ C_N(t+1) &= \alpha_N \cdot C_N(t) + \beta_N \cdot E_N(t) \end{aligned}$$

α_C and α_N close to 1, α_M closer to 0.

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\end{aligned}
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Radiative forcing

$$
\begin{aligned}
RF(t) &= \gamma_C \cdot \ln\left(C_C(t) / C_C(0)\right) \\
&\quad + \gamma_N \cdot \left(\sqrt{C_N(t)} - \sqrt{C_N(0)}\right) \\
&\quad + \gamma_M \cdot \left(\sqrt{C_M(t)} - \sqrt{C_M(0)}\right) \\
&\quad + \overline{RF}_X(t)
\end{aligned} \tag{12}
$$

where the γ's are constants and $\overline{RF}_X(t)$ from other gases and objects.

Albedo effects of land at time t will enter here.

Atmosphere

The environmental constraint reads

$$RF(t) \leq \overline{RF}(t)$$

for all t. We may also wish to go from radiative forcing to temperature, but that is left out for the moment.

Objective

The objective is to

$$\text{maximize} \sum_{t=0}^{T} \frac{1}{(1+r)^t} \left(CS(t) + PS(t) \right)$$

with respect to the choice variables

· fertilization $N(t)$,

· the amount of plants used to produce meat $Y_{vm}(t)$,

· land used for plants $L_v(t)$,

· land deforested $L_f(t, a)$ (dimension $\mathbb{R}^{T \times A}$) and

· land afforested $L_p(t)$

subject to all constraints.

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Consumer surplus food

$$CS(t) = \frac{1}{2}\left(\beta_v (X_v(t))^2 + \beta_a (X_a(t))^2\right)$$

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Producer surplus

$$\begin{aligned} PS(t) &= P_v(t) X_v(t) + P_a(t) X_a(t) + P_c(t) V(t) \\ &\quad - P_N(t) N(t) - q_C(t) - q_M(t) - q_N(t) \end{aligned}$$

where $P_c(t)$ and $P_N(t)$ are the unit prices of timber and fertilizers respectively, currently fixed.