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Noncooperative models of permit markets

by

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Abstract

The applicability of some popular and basic permit market theories has been questioned. Drawing on noncooperative equilibrium theory for pure exchange economies, this article adapts several well-established alternative models to permit exchange. Some qualitative properties of the associated equilibria are provided, including two games with equilibria that in a sense coincide. Nevertheless, as there exist quite a few models potentially applicable to emissions trading, with equilibria that range from autarky to Pareto optimality, it seems that economics lacks a broadly accepted basic theory for permit markets.

Keywords: Permit markets, market power, multilateral oligopoly, strategic exchange.

JEL classification: C72, D43, D51, Q58.

1 Introduction

Environmental economics has modeled permit markets for some time. Most studies either rely on Montgomery's [22] assumption that all agents are price takers, or follow Hahn [14] and Westskog [30] in allowing some participants to take dominant positions, as long as they are accompanied by a competitive fringe.

While criticism of perfect competition is longstanding and well known, the 'dominant agent competitive fringe' formulation has more recently been questioned in the context of permit markets, e.g. Godal [11], Malueg and Yates [18] and Wirl [31]. One problem arises because the fringe must be nonempty, yet there is no established

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and operational guide as to which agents should belong to it.[1] Because that choice may have substantial implications, the model appears in some sense incomplete.

Before embarking on research programs dedicated to formulating alternative theories of emissions exchange, it may be recalled that permit markets may be seen as a special case of standard exchange economies with only two 'goods', permits and money, and where utility is concave in the first good and linear in the second, i.e. quasilinear.[2] Therefore, most theories of pure exchange in principle apply directly to permit exchange. At first sight, this is encouraging, because much effort has been spent on modeling exchange economies, and there exists a large body of literature, most of which has had relatively little apparent impact on environmental and resource economics.

This article selects some models from the general literature on exchange economies modeled as noncooperative games and adapts them to emissions markets.[3] The models considered are all well established, most originating from the 1970s. Several of them are still active research programs in the general theory of pure exchange, and with one exception, all games discussed below have 'quantity' as the strategic variable.

In addition to defining the games, some qualitative properties of the associated equilibria are provided. These properties primarily relate to the comparison of marginal payoffs (i.e. marginal abatement costs) in equilibrium with the clearing price. It is demonstrated that in two games, all agents end up with identical marginal payoffs equal to the equilibrium price. In two other games, all agents will have a marginal payoff below or equal to the equilibrium price. In the remaining four games, marginal payoffs are below the equilibrium price for strategic sellers, and above it for strategic buyers. These results may perhaps suggest the models that could be eliminated via empirical observations—possibly generated in the lab.

We also show that one endowment-withholding game gives the same outcome as a technology misrepresentation game should technologies be quadratic. Also presented are some conditions under which autarky becomes the only equilibrium in a Shapley- and Shubik-type strategic market game. A simple, tractable example, which has been construed in view of studies of the carbon market as laid down by the Kyoto Protocol, illustrates the latter results.

Some items not discussed below may be noted right away. No extensions to,

[1] Misiolek and Elder [20, p. 159] suggest that if firms, when modeled as price takers end up with a large market share, then "it is interpreted as evidence that the market may be susceptible to manipulation". While this seems intuitive, one wonders where for practical purposes the line between 'large' and 'not large' should be drawn. Montero [21, Section 3.1] suggests that "the fringe must be rather large for the model to work well". Nevertheless, the definition of 'large' remains unspecified.

[2] More generally, this *setting* fits under the 'market game with transferrable payoff' umbrella, as for example in Osborne and Rubinstein [23, Section 13.4] and references therein. The *solution* concept discussed there, however, is the core.

[3] This application serves as motivation for this paper. Other applications for environmental and resource economics include the exchange of user rights to water and catch quotas for fish.

say, dynamics, uncertainty or interactions with other markets are considered.[4] In contrast, the focus of this study is deliberately and exclusively on a rather simple and arguably basic setting. Further, queries concerning the existence of equilibria, uniqueness, and possible convergence to perfect competition are left out, as are such issues as perspectives inspired by Bertrand-type competition, bargaining theory, auction theory, cooperative game theory, and out-of-equilibrium theory.

The article is organized as follows. Some preliminaries are detailed in Section 2. Section 3 defines eight distinct models of strategic exchange and presents some properties associated with their outcomes. Section 4 offers some special results on selected models and Section 5 summarizes and concludes.

2 Preliminaries

There is a fixed and finite set I of agents. Each $i \in I$ is already endowed with $e_i \geq 0$ units of permits, satisfying $\sum_{i \in I} e_i > 0$. When agent i has x_i available for himself, he incurs payoff $\pi_i(x_i)$. We shall not restrict π_i to any particular functional format before Section 4. Until then, we assume that $\pi_i'(\cdot) > 0$ and $\pi_i''(\cdot) < 0$ with $\pi_i'(x_i) \to \infty$ as $x_i \to 0$. The latter condition is merely included to avoid repeated assumptions on interior solutions.

Even though most of the theories discussed do not require agents to be price takers, all models can accommodate such behavior. Hence, for comparisons with the 'dominant agent competitive fringe' model we shall consider both strategists and price takers. More precisely, we say that an agent is a *price taker* if he consistently regards prices as parameters. Such agents, if any, are collected in a set F. Strategists, on the other hand, fully understand how their own actions affect prices, and belong to a complementary set $S := I \setminus F$.

In most models, each agent may act positively in both supply of and demand for permits. To avoid ambiguity, we therefore declare an agent to be a *seller* (*buyer*) if he ends up holding less (more) than his initial endowment, e_i.

We complete this section by introducing a problem for which we shall derive a comparative static result to be applied frequently. We denote a nonempty subset of I as $\mathcal{I}$ and consider the following problem

$$\max_{(x_i)_{i \in \mathcal{I}}} \left\{ \sum_{i \in \mathcal{I}} \pi_i(x_i) : \sum_{i \in \mathcal{I}} x_i = Q \right\}. \tag{1}$$

As is well known, when $Q = \sum_{i \in \mathcal{I}} e_i$, and $\mathcal{I} = I$, with the shadow (clearing) price p associated with the resource balance constraint, the solution to this problem coincides with that of a perfectly competitive permit market, being a vector $(x_i)_{i \in I}$ and clearing price p where x_i maximizes $\pi_i(x_i) + p \cdot (e_i - x_i)$ for each $i \in I$ satisfying $\sum_{i \in I} x_i = \sum_{i \in I} e_i$.

[4] For a discussion of some of these extensions, see Montero [21].

It is convenient to introduce problem (1) even when modeling imperfect competition. This is because noncooperative models of exchange typically include a competitive element at a second game stage, on a subset of either agents or decision variables. Moreover, for all but one of the models, prices emerge as Lagrangian multipliers. Modified versions of problem (1) will therefore appear throughout, instead of listing all individuals' maximization problems together with the market clearing condition.

Concerning problem (1), we note first that under the assumed conditions, the optimal allocation of permits is interior and unique, as is the clearing price. Moreover, it is of particular interest how resulting demand x_i of individual i as well as clearing price p will be affected when supply Q is perturbed. With apologies for abusing notation, I denote these derivatives as $\frac{\partial p}{\partial Q}$ and $\frac{\partial x_i}{\partial Q}$.

Lemma 2.0 *In problem* (1) *and with the assumed properties on* $\pi_i(\cdot)$*, we have that* $\frac{\partial p}{\partial Q}$ *and* $\frac{\partial x_i}{\partial Q}$ *exist and are characterized by*

$$\frac{\partial p}{\partial Q} = \frac{1}{\sum_{i\in\mathcal{I}} \frac{1}{\pi_i''(x_i)}} < 0 \text{ and } \frac{\partial x_i}{\partial Q} = \frac{1}{\pi_i''(x_i)} \frac{\partial p}{\partial Q} \in (0,1) \text{ for each } i \in \mathcal{I}. \quad (2)$$

These results confirm the intuition and established results.[5] Prices fall with increasing supply and if one extra unit is made available at the market, agent i will take part of it home.

3 Eight models of permit exchange

3.1 Dominant agents with a competitive fringe

We start with the most commonly applied model for strategic permit exchange, for which Hahn [14] and Westskog [30] are standard references. Interaction has a two-stage nature whereby each strategist $i \in S$ first chooses the amount x_i he wants to retain for himself by solving

$$\max_{x_i} \{\pi_i(x_i) + p \cdot (e_i - x_i)\}$$

recognizing that p will depend on his own x_i. For ease of exposition, this dependence is tacitly understood and will notationally be suppressed—here and in similar instances. At the second stage of the game the *nonempty* fringe F allocates

$$Q := \sum_{i\in I} e_i - \sum_{i\in S} x_i$$

via perfect competition. Hence, as in (1), the fringe acts as if it solves

$$\max_{(x_i)_{i\in F}} \left\{ \sum_{i\in F} \pi_i(x_i) : \sum_{i\in F} x_i = Q \right\}$$

[5] Those readers interested in more general comparative statics results may consult [7].

with p as the associated price. Thus, overall equilibrium, granted it exists, is characterized by

$$\left.\begin{array}{l}\pi_i'(x_i) = p \text{ for all } i \in F,\ \sum_{i\in F} x_i = Q, \text{ and} \\ \pi_i'(x_i) + \frac{\partial p}{\partial x_i} \cdot (e_i - x_i) = p \text{ for all } i \in S\end{array}\right\} \tag{3}$$

where by the same argument as in Lemma 2.0, it follows that

$$\frac{\partial p}{\partial x_i} = \frac{-1}{\sum_{i\in F} \frac{1}{\pi_i''(x_i)}} > 0 \tag{4}$$

for each $i \in S$. (4) says that the more permits a strategic agent keeps for himself, the fewer become available for the fringe, and the higher is the equilibrium price.

Conditions (3)–(4) demonstrate the following directly.

Proposition 3.1
• *A price taker has a marginal payoff that equals the equilibrium price.*
• *A strategic seller (buyer) has a marginal payoff that is below (above) the equilibrium price.*

Whereas these statements seem reasonable and in line with commonplace economic jargon, the model is silent concerning how agents should be classified as strategists and price takers. Because the fringe must be nonempty for the game to be well defined, that choice cannot be avoided, and—as alluded to in the Introduction—it may have important and somewhat discouraging implications, see e.g. Godal [11], Montero [21] and Wirl [31].

3.2 Endowment destruction

One way of gaming an exchange market is to destroy some endowment before engaging in trade, e.g. Aumann and Peleg [1], Guesnerie and Laffont [13], Mas-Colell [19] and Postlewaite [25, D-manipulation]. That is, each agent $i \in I$ decides to keep amount $q_i \in [0, e_i]$ intact. The chosen q_i is brought to the market, and derives from the first-stage problem

$$\max_{q_i} \left\{\pi_i(x_i) + p \cdot (q_i - x_i)\right\}. \tag{5}$$

In (5), demand x_i and price p, which both depend on $\sum_{i\in I} q_i$, are settled by

$$\max_{(x_i)_{i\in I}} \left\{\sum_{i\in I} \pi_i(x_i) : \sum_{i\in I} x_i = \sum_{i\in I} q_i\right\},$$

where, at this second stage, supply $\sum_{i\in I} q_i$ is taken as given and p is the associated shadow price. The difference $e_i - q_i$ for each $i \in I$ is understood to be destroyed.

Any Nash equilibrium in the overall game is characterized by

$$\left.\begin{array}{l}\pi_i'(x_i) = p \text{ for all } i \in I,\ \sum_{i\in I} x_i = \sum_{i\in I} q_i, \text{ and} \\ \pi_i'(x_i) \cdot \frac{\partial x_i}{\partial q_i} + p \cdot (1 - \frac{\partial x_i}{\partial q_i}) + \frac{\partial p}{\partial q_i} \cdot (q_i - x_i) - \lambda_i + \mu_i = 0 \text{ for all } i \in I,\end{array}\right\} \tag{6}$$

where $\lambda_i \geq 0$ is associated with $q_i \leq e_i$ and $\mu_i \geq 0$ with $q_i \geq 0$.

In (6), a strategist $i \in S$ foresees, by Lemma 2.0, that

$$\frac{\partial p}{\partial q_i} = \frac{1}{\sum_{j \in I} \frac{1}{\pi_j''(x_j)}} < 0 \text{ and } \frac{\partial x_i}{\partial q_i} = \frac{1}{\pi_i''(x_i)} \frac{\partial p}{\partial q_i} \in (0, 1),$$

whereas a price taker $i \in F$, if any, behaves as if these derivatives are nil. Two properties of this model follow.

Proposition 3.2
- *All agents have a final marginal payoff that equals the equilibrium price.*
- *All price takers and strategic buyers will always keep endowments intact.*

Whereas it is intuitive that someone who ends up being a buyer will never destroy any endowment, we see that for a given total supply the equilibrium will be efficient. Whether overall efficiency will result in no endowments being destroyed becomes dependent on parameters. For permit markets such as the one under the Kyoto agreement, where a substantial body of literature has predicted a competitive permit price close to zero, destruction may well take place in an equilibrium of this game.

3.3 Endowment withholding (I) with constrained supply

If what is kept away from the market may be used constructively, as opposed to destroyed, then we may discuss manipulation via withholding. Several versions will be discussed, beginning with Postlewaite's [25, W-manipulation]. Here, each $i \in I$ decides first how much $q_i \in [0, e_i]$ to bring to the market by solving

$$\max_{q_i} \{\pi_i (e_i - q_i + x_i) + p \cdot (q_i - x_i)\}. \tag{7}$$

At the second market stage, demand x_i and shadow/clearing price p come about from problem

$$\max_{(x_i)_{i \in I}} \left\{ \sum_{i \in I} \pi_i (x_i) : \sum_{i \in I} x_i = \sum_{i \in I} q_i \right\}. \tag{8}$$

so that x_i and p will depend on $\sum_{i \in I} q_i$.

An overall Nash equilibrium is characterized by

$$\left. \begin{array}{l} \pi_i'(x_i) = p \text{ for all } i \in I, \sum_{i \in I} x_i = \sum_{i \in I} q_i, \text{ and} \\ \pi_i'(e_i - q_i + x_i) \cdot \left(-1 + \frac{\partial x_i}{\partial q_i}\right) + p \cdot (1 - \frac{\partial x_i}{\partial q_i}) + \frac{\partial p}{\partial q_i} \cdot (q_i - x_i) - \lambda_i + \mu_i = 0 \end{array} \right\} \tag{9}$$

for all $i \in I$, where $\lambda_i, \mu_i \geq 0$ are the shadow prices associated with $q_i \leq e_i$ and $q_i \geq 0$, respectively. Once again, when solving problem (7), a strategist $i \in S$

behaves as though

$$\frac{\partial p}{\partial q_i} = \frac{1}{\sum_{j\in I} \frac{1}{\pi_j''(x_j)}} < 0 \text{ and } \frac{\partial x_i}{\partial q_i} = \frac{1}{\pi_i''(x_i)} \frac{\partial p}{\partial q_i} \in (0,1) \tag{10}$$

confer Lemma 2.0, whereas a price taker $i \in F$ treats both these objects as equal to zero.

The overall game will yield an inefficient outcome, as strategic sellers will withhold supply.

Proposition 3.3
• *Price takers and strategic buyers supply precisely their endowment to the marketplace and have a marginal payoff that equals the equilibrium price.*
• *A strategic seller supplies strictly less than his endowment, and has a final marginal payoff below the equilibrium price.*

Hence, and as with the endowment destruction model, a strategic agent who ends up being a buyer cannot do better than acting as a price taker. That changes in the next game considered.

3.4 Endowment withholding (I) with free supply

Safra [26] examines the last-mentioned model with one minor, yet important difference, in admitting $q_i > e_i$ as a feasible choice. Everything else is as in the previous model of endowment withholding, and equilibrium is characterized by (9) with $\lambda_i = 0$ for all $i \in I$. This modification opens the way for strategic buyers to act differently from price takers:

Proposition 3.4
• *A price taker brings precisely his endowment to the marketplace and has a final marginal payoff that equals the equilibrium price.*
• *A strategic seller (buyer) supplies less (more) than his endowment to the marketplace, and has a final marginal payoff that is below (above) the equilibrium price.*

The intuition this time is that a strategist who comes forward as a buyer attempts to 'push' prices down as if 'flooding' the market. Of course, in final consumption this upward misrepresentation of endowments must be accounted for, so that his final marginal payoff will be higher than the equilibrium price. On the other side of the market, strategic sellers hold back on supply to induce the opposite effect on prices.

3.5 Endowment withholding (II) with constrained demand

The model discussed here appears to originate from Codognato and Gabszewicz [3] and has been baptized 'exchange à la Cournot-Walras'. While further developed and examined in Bonnisseau and Florig [2], Gabszewicz and Michel [9] and Lahmandi-Ayed [17] among others, the exposition in this study follows the one given by Gabszewicz [8, Section 4.4].

Again, the game comes from a situation where each $i \in I$ first decides on how much $q_i \in [0, e_i]$ to bring to the market. As in the previous two models, this decision solves

$$\max_{q_i} \{\pi_i (e_i - q_i + x_i) + p \cdot (q_i - x_i)\} \tag{11}$$

where x_i is demand in the marketplace. At the second market stage, agents behave consistently with possibly having some remaining endowment at home. Their demand and the clearing price solve

$$\max_{(x_i)_{i \in I}} \left\{ \sum_{i \in I} \pi_i (e_i - q_i + x_i) : \sum_{i \in I} x_i = \sum_{i \in I} q_i \right\} \tag{12}$$

where again supply $\sum_{i \in I} q_i$ is taken as a datum and p is the associated shadow price.

Note that the assumption $\pi_i' (y_i) \to \infty$ as $y_i \to 0$ does not make the constraint $x_i \geq 0$ superfluous, because x_i is not the only argument of the payoff function in (12). Although constraints on decision variables at the first stage of the game are easily handled, it complicates matters when they may come into effect in the second stage. This makes the overall game less easily characterized than the previous ones. Nevertheless, because every agent has an objective function at the second stage of the game that is identical to that at the first stage, it is possible to make some statements about an overall Nash equilibrium even when characterizing the necessary conditions at the second stage only. There, it must hold that

$$\left. \begin{array}{l} \pi_i'(e_i - q_i + x_i) - \mu_i = p, \text{ and} \\ x_i \geq 0,\ \mu_i \geq 0,\ x_i \mu_i = 0 \text{ for all } i \in I, \text{ together with} \\ \sum_{i \in I} x_i = \sum_{i \in I} q_i. \end{array} \right\} \tag{13}$$

Proposition 3.5 *Suppose x_i must be nonnegative. Then, every permit buyer has a marginal payoff that equals the equilibrium price, and no agent has a marginal payoff above the equilibrium price.*

This game therefore has some of the same qualitative properties as the endowment withholding game with constrained supply—see Proposition 3.3.

3.6 Endowment withholding (II) with free demand

The above game is next modified by allowing q_i to be negative, as d'Aspremont et al. [5, p. 203] do. To guarantee market clearing, we therefore also allow for demand

$x_i < 0$. Although negative supply and demand may not be appealing in reality, one may perhaps think of these items as messages rather than physical quantities. However they are interpreted, we obtain the following result.

Proposition 3.6 *Suppose $x_i < 0$ is an admissible choice. Then, all agents have the same final marginal payoff, which equals the equilibrium price. That is the unique competitive price.*

Therefore, this game has the notable property that Nash equilibria are Pareto efficient. The explanation is simple. Whatever is supplied at the first stage of the game, permits flow freely among parties at the second stage until all agents have the same margin. Moreover, and as with the endowment destruction model above, the allocation at the second stage is the same as at the first. However, because all resources are intact in this game, Nash equilibria become perfectly competitive.

3.7 Manipulation via technologies

Here we present a game that, in the context of pure exchange, dates back at least to Hurwicz [16, Footnote 10]. Shin and Suh [28], Malueg and Yates [18], Wirl [31], and Godal and Meland [12] have applied it to permit markets. It also seems to fit the 'supply function equilibrium' terminology, as discussed by Hendricks and McAfee [15], for example. As we shall compare this model with another later on, some new notations are introduced.

Here, each agent $i \in I$ first selects a payoff function, say $\hat{\pi}_i : \mathbb{R}_+ \rightarrow \mathbb{R}$, which solves

$$\max\{\pi_i(y_i) + r \cdot (e_i - y_i)\}.$$

where r is the permit price. At the second stage, endowments are traded competitively with respect to the chosen technologies, generating an allocation $(y_i)_{i \in I}$ that

$$\text{maximizes} \left\{ \sum_{i \in I} \hat{\pi}_i(y_i) : \sum_{i \in I} y_i = \sum_{i \in I} e_i \right\}$$

with r clearing the market. To obtain some qualitative results for this game, we next consider the format where the choice $\hat{\pi}_i$ is constrained to the 'quadratic' case, i.e. that $\hat{\pi}_i'(y_i) := \max\{a_i - b_i y_i, 0\}$ where $a_i, b_i > 0$.

Suppose there exists an equilibrium in this game satisfying $\sum_{i \in I} \frac{a_i}{b_i} > \sum_{i \in I} e_i$, therefore characterized by

$$(\pi_i'(y_i) - r)\frac{\partial y_i}{\partial a_i} + \frac{\partial r}{\partial a_i}(e_i - y_i) = 0 \tag{14}$$

and

$$(\pi_i'(y_i) - r)\frac{\partial y_i}{\partial b_i} + \frac{\partial r}{\partial b_i}(e_i - y_i) = 0 \tag{15}$$

together with the second-stage conditions, which may be written as

$$y_i = (a_i - r)/b_i \text{ for all } i \in I, \text{ and } r = \frac{\sum_{i\in I} \frac{a_i}{b_i} - \sum_{i\in I} e_i}{\sum_{i\in I} \frac{1}{b_i}}. \tag{16}$$

What remains to spell out is precisely how y_i and r are affected by changes in a_i and b_i. By differentiating the two equalities in (16), one obtains, with some rearrangements, that

$$\frac{\partial r}{\partial a_i} = \frac{\frac{1}{b_i}}{\sum_{j\in I} \frac{1}{b_j}} \in (0,1), \quad \frac{\partial y_i}{\partial a_i} = \frac{1}{b_i}\left(1 - \frac{\partial r}{\partial a_i}\right) > 0, \tag{17}$$

$$\frac{\partial r}{\partial b_i} = -y_i \frac{\frac{1}{b_i}}{\sum_{j\in I} \frac{1}{b_j}} < 0 \quad \text{and} \quad \frac{\partial y_i}{\partial b_i} = -\frac{1}{b_i}\left(y_i + \frac{\partial r}{\partial b_i}\right) < 0 \tag{18}$$

for every strategist $i \in S$. Because a price taker by definition believes that $\frac{\partial r}{\partial a_i} = \frac{\partial r}{\partial b_i} = 0$, it follows from (17)–(18) that

$$\frac{\partial y_i}{\partial a_i} = \frac{1}{b_i} > 0 \text{ and } \frac{\partial y_i}{\partial b_i} = -\frac{y_i}{b_i} < 0 \tag{19}$$

for each $i \in F$, if any. The next result follows directly by applying the signs of the various objects in (17)–(19) into (14)–(15).

Proposition 3.7 *Suppose there exists an equilibrium where $a_i, b_i > 0$ for all $i \in I$ and $\sum_{i\in I} \frac{a_i}{b_i} > \sum_{i\in I} e_i$. Then,*
- *a price taker has a marginal payoff that equals the equilibrium price;*
- *a strategic seller (buyer) has a marginal payoff that is below (above) the equilibrium price.*

It is worth noting by (18), that (15) equals (14) multiplied by $-y_i$ throughout. Hence, if a pair (a_i, b_i) satisfy (14), then (15) is automatically granted.

Some special cases of this game have been applied in various ways to emissions exchange, all assuming that the true benefit function is quadratic with margin $\pi_i'(y_i) = \max\{\alpha_i - \beta_i y_i, 0\}$, where $\alpha_i, \beta_i > 0$. Specifically, Malueg and Yates [18] study a situation where β_i is the same for all, and where only α_i is gamed. Godal and Meland [12] consider the same case, although allowing for β_i to vary across i. Wirl [31] supposes that marginal benefit function given by $\pi_i'(y_i) = \max\{\beta_i\gamma - \beta_i y_i, 0\}$, where $\gamma > 0$. In his game, β_i may be misrepresented.

3.8 The trading post model

Finally, we shall consider the trading post model of Shapley and Shubik [27] for which the term 'strategic market game' has been reserved.[6] This model appears to

[6] See Giraud [10] for an introduction to a special issue on this game.

be the most popular one for more general pure exchange economies and there exist many versions of it. The one adopted here goes as follows. Each agent $i \in I$ places $q_i \in [0, e_i]$ units of permits and $b_i \geq 0$ units of money on a 'trading post'. Suppose each agent has enough money so that no upper bounds on b_i come into effect. That is, money is in what is known as 'sufficient supply'.[7] Name aggregate *supply* and *bid*

$$Q := \sum_{i \in I} q_i \text{ and } B := \sum_{i \in I} b_i,$$

respectively, and consider first the case $B, Q > 0$. Then, trade occurs at the unit price

$$p := \frac{B}{Q} \tag{20}$$

and agent $i \in I$ is paid pq_i for his permit supply and takes home $\frac{b_i}{p}$ permits from the post. Should $B = 0$, then we assume that whatever that has been supplied, if anything, is lost, and similarly for any positive bids should $Q = 0$.

Whenever $B, Q > 0$, each agent $i \in I$ selects a pair (q_i, b_i) that

$$\text{maximizes}\{\pi_i(e_i - q_i + \frac{b_i}{p}) + pq_i - b_i\}. \tag{21}$$

Write $\Pi_i(\cdot)$ for the objective function in (21), so that

$$\frac{\partial \Pi_i}{\partial b_i} = \pi_i'(e_i - q_i + \frac{b_i}{p})\frac{p - b_i \frac{\partial p}{\partial b_i}}{p^2} + \frac{\partial p}{\partial b_i} q_i - 1 \tag{22}$$

and

$$\frac{\partial \Pi_i}{\partial q_i} = \pi_i'(e_i - q_i + \frac{b_i}{p})\left(-1 - \frac{b_i \frac{\partial p}{\partial q_i}}{p^2}\right) + p + \frac{\partial p}{\partial q_i} q_i - \lambda_i. \tag{23}$$

where λ_i is the shadow price associated with $q_i \leq e_i$. The necessary first-order optimality conditions therefore read

$$\frac{\partial \Pi_i}{\partial b_i} \leq 0,\ b_i \geq 0 \text{ and } \frac{\partial \Pi_i}{\partial b_i} b_i = 0; \tag{24}$$

$$\frac{\partial \Pi_i}{\partial q_i} \leq 0,\ q_i \geq 0 \text{ and } \frac{\partial \Pi_i}{\partial q_i} q_i = 0; \tag{25}$$

$$\lambda_i \geq 0,\ q_i \leq e_i \text{ and } (q_i - e_i)\lambda_i = 0 \tag{26}$$

for all $i \in I$. Every strategist $i \in S$ behaves consistently with setting

$$\frac{\partial p}{\partial b_i} = \frac{1}{Q} \text{ and } \frac{\partial p}{\partial q_i} = -\frac{p}{Q} \tag{27}$$

[7] In some sense, one may say that this is an underlying assumption in the other models discussed in this paper as well.

whereas these objects vanish for a price taker $i \in F$.

Proposition 3.8 *Suppose there exists an equilibrium with at least two suppliers and two bidders, i.e. that $b_i < B$ and $q_i < Q$ for all $i \in I$. Then,*
• *a price taker has a marginal payoff that equals the equilibrium price;*
• *a strategic seller (buyer) has a marginal payoff that is below (above) the equilibrium price.*

As is clear and well known, the profile $(q_i, b_i) = (0, 0)$ for all $i \in I$, is one equilibrium in this game.

4 Selected models and special results

This section presents two results for special environments, under the assumption that all agents act strategically. We start by demonstrating that if payoffs are quadratic, then the endowment withholding game, as detailed in Section 3.4 above, produces the same equilibrium as a special version of the payoff manipulation game in Section 3.7 up to first-order optimality conditions. Next, if there is plenty of 'hot air' (to be defined) in the economy, then the trading post model in Section 3.8 has no equilibrium with trade. The section concludes with an illustrative example.

4.1 A first-order equivalency result

Our first result considers the following setting.

Assumption 4.1
• *Marginal payoff functions are given by $\pi_i'(x_i) = \max\{\alpha_i - \beta_i x_i, 0\}$ for all $i \in I$.*
• *In the manipulation via technology game of Section 3.7, all agents may misrepresent α_i by a_i, whereas b_i is fixed to the true β_i. Further, there exists an equilibrium in that game satisfying $a_i > 0$ for all $i \in I$ and $\sum_{i \in I} \frac{a_i}{\beta_i} > \sum_{i \in I} e_i$.*
• *There exists a Nash equilibrium profile $(q_i)_{i \in I}$ in the endowment withholding game* (I) *with free supply (as defined in Section 3.4) satisfying $\sum_{i \in I} q_i < \sum_{i \in I} \alpha_i / \beta_i$, with a resulting demand profile $(x_i)_{i \in I}$ and clearing price p generating the final allocation $(e_i - q_i + x_i)_{i \in I}$.*

Proposition 4.1 *Given Assumption 4.1, then the strategy $a_i := \alpha_i + \beta_i (e_i - q_i)$ for all $i \in I$ satisfies all necessary first-order optimality conditions in the technology manipulation game. This profile of choices generates the final allocation of permits, $y_i = e_i - q_i + x_i$ for all $i \in I$, as well as the clearing price $r = p$.*

To have a genuine equivalency result, it appears that one would need to deal with the existence and uniqueness of equilibria in the two games. Although that is not addressed here, it seems that $\pi_i(x_i)$, being quadratic, will promote such properties.

One may wonder whether the above result generalizes to environments where $\pi_i(x_i)$ is not quadratic. The answer to this question appears somewhat negative, because the formulas for $\frac{\partial p}{\partial q_i}$ and $\frac{\partial x_i}{\partial q_i}$ as given in (10) depend on the second derivative of the payoff functions evaluated at the *interim* allocation x_i for all $i \in I$; by contrast, in the technology manipulation game, they are evaluated at the *final* allocation y_i for all $i \in I$. Because x_i will typically differ from y_i and as the proof depends critically on the property that $\pi_i''(x_i) = \pi_i''(y_i)$, there is a certain 'necessity' to the 'quadratic' restriction.

4.2 A no-trade result for trading posts

For the next result, some terminology needs to be clarified.

Definition 4.2
• *Business-as-usual emissions* $\hat{x}_i$, is a strictly positive finite number for which $\pi_i(x_i) < \pi_i(\hat{x}_i)$ when $x_i \in [0, \hat{x}_i)$ and $\pi_i(x_i) = \pi_i(\hat{x}_i)$ when $x_i \geq \hat{x}_i$.
• An agent has *hot air* if $e_i > \hat{x}_i$, whereas an agent is *short* if $e_i < \hat{x}_i$.
• The economy has *hot air in aggregate* if $\sum_{i \in I} e_i > \sum_{i \in I} \hat{x}_i$.

For instance, the payoff functions considered in the previous subsection exhibit business-as-usual emissions $\hat{x}_i = \alpha_i / \beta_i$. Nevertheless, in what follows we shall not restrict our attention to that particular functional format.

Assumption 4.2
• $\pi_i(\cdot)$ is nondecreasing, concave and continuously differentiable for each $i \in I$;
• there exist *business-as-usual emissions* $\hat{x}_i > 0$ for all $i \in I$;
• there is at least one agent who is *short* and at least two agents with *hot air*; and
• there is *hot air in aggregate.*

The first part of the third bullet point only serves to provide an interesting economy where autarky is Pareto inefficient. The concavity assumption in the first bullet point is never explicitly used, but guarantees the existence of a competitive equilibrium. Note that under the stated conditions, such an equilibrium entails trade at a vanishing price.

Proposition 4.2 *Given Assumption 4.2, then a Nash equilibrium with trade does not exist in the Shapley–Shubik strategic market game (Section 3.8).*

Roughly speaking, the main mechanism at work for the result is the following. Suppose in contrast that several agents have offered strictly positive supplies, q_i, and bids, b_i, generating trade. Then there will be at least one agent that has an incentive to reduce his bid. This holds no matter how little he bids. Therefore, he will not bid. A consequence of this is that there will be some other agent with an incentive to reduce his bid, and so forth. In the spirit of induction, this will spread throughout the economy so that all bids vanish, and with them, trade.

It must be emphasized that satiation in payoffs—i.e. that business-as-usual emissions are finite—represents a violation of standard assumptions in general equilibrium theory, including those adopted by Peck et al. [24], where sufficient conditions for equilibria with trade to exist are provided. Other conditions for autarky to become the only equilibrium than those specified above, are given in Cordella and Gabszewicz [4].[8] These issues have more recently been discussed in Dickson and Hartley [6].

4.3 An example

Here is an example that illustrates the results in this section in a tractable manner. A more interesting one, which generates the same qualitative results for carbon trading under the Kyoto agreement, may be found in Godal and Meland [12, Table 1].[9] There are four strategic agents, $i = 1, 2, 3, 4$, all with $\pi_i'(x_i) = \max\{100 - x_i, 0\}$, and where the initial allocation $(e_i)_{i \in I} = (90, 90, 120, 120)$.

Starting with the endowment withholding game (I) with free supply in Section 3.4, a profile $(q_i, x_i)_{i \in I}$ with a clearing price $p = 2$ that satisfies all the necessary first-order optimality conditions for the example is listed next.

Agent, i	1,	2,	3,	4	Total
Supply, q_i	92,	92,	104,	104	392
Demand, x_i	98,	98,	98,	98	392
Final allocation, $e_i - q_i + x_i$	96,	96,	114,	114	420
Marg. payoff, $\pi_i'(e_i - q_i + x_i)$	4,	4,	0,	0	

Thus, the second part of Proposition 3.4 is illustrated.

Consider next the technology misrepresentation game in Section 3.7, where we fix $b_i = \beta_i (= 1)$ for all agents. A profile $(a_i, y_i)_{i \in I}$ with the clearing price $r = 2$ that satisfies the associated first-order optimality conditions is given as follows.

[8] Their result is based on an economy where preferences are linear. Further, if their economy is replicated sufficiently many times, equilibria with trade will eventually exist. In our economy, autarky prevails as the only equilibrium, no matter how many times the economy is replicated.

[9] When it comes to emissions trading under the Kyoto agreement, on which there is a large body of literature dealing with numerical simulations, it is well known that hot air is present in Russia and the Ukraine, among others. In addition, several studies have suggested that without US participation, there is hot air in aggregate (i.e. a competitive price that vanishes), see, for example, Springer [29] for an overview. Therefore, the no-trade result in Proposition 4.2 will also apply to other parameterizations of the Kyoto setting.

Agent, i	1,	2,	3,	4	Total
Technology, a_i	98,	98,	116,	116	
Final allocation, y_i	96,	96,	114,	114	420
Marg. payoff, $\pi_i'(y_i)$	4,	4,	0,	0	

This illustrates the second statement in Proposition 3.7 as well as Proposition 4.1.

Finally, business-as-usual emissions for each agent in the example equal 100 units. Therefore, agents 3 and 4 have so much hot air that this also applies in aggregate. Hence, the example satisfies Assumption 4.2, yielding the no-trade result in Proposition 4.2 for the trading post model.

5 Summary and concluding remarks

Revisited above were several well-established models of noncooperative exchange that could possibly apply to emissions exchange. In terms of the qualitative properties of the associated equilibria, they may be grouped into three: first, models where all agents have the same marginal payoff equal to the equilibrium price (Sections 3.2 and 3.6); second, models where all agents have a marginal payoff below or equal to the equilibrium price (Sections 3.3 and 3.5); and third, those that are compatible with marginal payoffs in equilibrium below the equilibrium price for strategic sellers and above the price for strategic buyers, as in Sections 3.1, 3.4, 3.7 and 3.8. Moreover, sufficient conditions have been provided for the games in Sections 3.4 and 3.7 to yield the same outcomes, and for the strategic market game in Section 3.8 to have no equilibrium with trade.

Any reader seeking published criticism or appraisal of a particular model is likely to find it; see, for example, Godal and Meland [12, Section 6] for a collection of passages. Given all the models and the diversity in the outcomes they produce, it seems to me that whether one is interested in consumers exchanging apples for oranges, or producers trading permits for money, economics has not yet come up with a broadly accepted theory for exchange economies.

APPENDIX: Proofs

Proof of Lemma 2.0. As $\pi_i'(x_i) = p$, and $\pi_i''(x_i) < 0$, there exist a continuously differentiable demand function $f_i := (\pi_i')^{-1}$ such that $x_i = f_i(p)$ for each $i \in \mathcal{I}$. By making use if the Inverse Function Theorem, we get $f_i' = \frac{1}{\pi_i''}$. Market clearing requires $\sum_{i\in\mathcal{I}} f_i(p) = Q$. It therefore follows by differentiating the last equality with respect to Q that $\frac{\partial p}{\partial Q}\sum_{i\in\mathcal{I}} \frac{1}{\pi_i''(x_i)} = 1$, which gives the first part of (2). The second statement follows by differentiating $x_i = f_i(p)$ with respect to Q. □

Proof of Proposition 3.2. Because final consumption equals demand, the first statement follows directly from (6). We obtain the second statement by combining the two lines in (6) that

$$\pi_i'(x_i) + \frac{\partial p}{\partial q_i}\cdot(q_i - x_i) - \lambda_i + \mu_i = 0 \tag{28}$$

for all $i \in I$. If $\mu_i > 0$, so that $q_i = 0$ and $\lambda_i = 0$, then (28) reads $\pi_i'(x_i) - \frac{\partial p}{\partial q_i}x_i + \mu_i = 0$, which is impossible as $\pi_i'(x_i) > 0$, $\frac{\partial p}{\partial q_i} < 0$ and $x_i \geq 0$. Hence, $\mu_i = 0$. Now, if agent i is a price taker, it follows immediately from (28) that $\lambda_i > 0$ because $\frac{\partial p}{\partial q_i}$ is seen as zero, and $\pi_i'(x_i) > 0$. Thus $q_i = e_i$. The same result follows for a strategic agent with $(q_i - x_i) < 0$ (i.e. a buyer), as $\frac{\partial p}{\partial q_i} < 0$. □

Proof of Proposition 3.3. We start by showing that μ_i must $= 0$. Suppose conversely that $\mu_i > 0$, yielding $q_i = 0$ and $\lambda_i = 0$. Then, by using the first equality in (9), the second line in the same statement reads

$$\left(\pi_i'(e_i + x_i) - \pi_i'(x_i)\right)\cdot\left(-1 + \frac{\partial x_i}{\partial q_i}\right) - \frac{\partial p}{\partial q_i}\cdot x_i + \mu_i = 0. \tag{29}$$

As $e_i \geq 0$, π_i' is strictly decreasing, $\frac{\partial x_i}{\partial q_i} \in (0,1)$, $\frac{\partial p}{\partial q_i} < 0$ and $x_i \geq 0$, the left-hand side of (29) is strictly positive, a contradiction making $\mu_i = 0$.

Consider now first a price taker who sees $\frac{\partial p}{\partial q_i} = \frac{\partial x_i}{\partial q_i} = 0$ and assume conversely that he chooses $q_i < e_i$ so that $\lambda_i = 0$. We then obtain from (9) the contradiction that

$$0 = -\pi_i'(e_i - q_i + x_i) + p = -\pi_i'(e_i - q_i + x_i) + \pi_i'(x_i) > 0$$

as π_i' is strictly decreasing and $q_i < e_i$. Thus, a price taker supplies exactly his endowment, and the first claim in the first bullet point is proved. We turn next to a strategic buyer and make the converse assumption that $q_i < e_i$ so that $\lambda_i = 0$. Rearrange (9) and make use of $\mu_i = 0$ to obtain the contradiction

$$\begin{aligned} 0 &= \left(p - \pi_i'(e_i - q_i + x_i)\right)\cdot\left(1 - \frac{\partial x_i}{\partial q_i}\right) + \frac{\partial p}{\partial q_i}\cdot(q_i - x_i) \\ &= \left(\pi_i'(x_i) - \pi_i'(e_i - q_i + x_i)\right)\cdot\left(1 - \frac{\partial x_i}{\partial q_i}\right) + \frac{\partial p}{\partial q_i}\cdot(q_i - x_i) > 0 \end{aligned}$$

as $\pi_i'(x_i) = p$, π_i' is strictly decreasing, $\frac{\partial x_i}{\partial q_i} \in (0,1)$, $\frac{\partial p}{\partial q_i} < 0$ and $q_i < x_i$, because he is a buyer. Hence, he supplies precisely his endowment and the rest of the first claim is proved.

For the second bullet point, suppose on the contrary that $q_i = e_i$. That yields the contradiction

$$\begin{aligned} 0 &= (p - \pi_i'(e_i - q_i + x_i)) \cdot \left(1 - \frac{\partial x_i}{\partial q_i}\right) + \frac{\partial p}{\partial q_i} \cdot (q_i - x_i) - \lambda_i \\ &= (\pi_i'(x_i) - \pi_i'(e_i - q_i + x_i)) \cdot \left(1 - \frac{\partial x_i}{\partial q_i}\right) + \frac{\partial p}{\partial q_i} \cdot (q_i - x_i) - \lambda_i \\ &= \frac{\partial p}{\partial q_i}(q_i - x_i) - \lambda_i < 0 \end{aligned}$$

because $q_i = e_i$, $\frac{\partial p}{\partial q_i} < 0$, $q_i > x_i$ and $\lambda_i \geq 0$. Therefore, he supplies strictly less than his endowment, and because $\pi_i'(x_i) = p$ and π_i' is strictly decreasing, it follows that $\pi_i'(e_i - q_i + x_i) < p$. □

Proof of Proposition 3.4. The claims are proved by the same type of arguments as in the proof of Proposition 3.3. □

Proof of Proposition 3.5. As $q_i \geq 0$ and agent i is a net buyer, i.e. $x_i > q_i$, the constraint $x_i \geq 0$ cannot bite. Thus, from the first line in (13), $\pi_i'(e_i - q_i + x_i) = p$ for any buyer regardless of whether he is strategic or not. The last claim follows trivially as $\mu_i \geq 0$. □

Proof of Proposition 3.6. Because x_i is a free variable, the shadow price μ_i disappears. Thus, (13) yields $\pi_i'(e_i - q_i + x_i) = p$ for all $i \in I$. By writing $f_i := (\pi_i')^{-1}$ we get $e_i - q_i + x_i = f_i(p)$. Inserting this in the market clearing condition, it follows that $\sum_{i \in I}(f_i(p) - e_i + q_i) = \sum_{i \in I} q_i$, i.e. that $\sum_{i \in I} f_i(p) = \sum_{i \in I} e_i$. Clearly, the price p that solves this equation is the perfectly competitive one, which under the assumed conditions is unique. □

Proof of Proposition 3.7. Follows directly from the main text.

Proof of Proposition 3.8. Concerning the first bullet point, suppose first that $\frac{\partial \Pi_i}{\partial b_i} = 0$, then the statement follows simply by inserting $\frac{\partial p}{\partial b_i} = 0$ into (22). Next, suppose alternatively that $\frac{\partial \Pi_i}{\partial b_i} = \pi_i'(e_i - q_i + \frac{b_i}{p})\frac{1}{p} - 1 < 0$, which implies $b_i = 0$, yielding

$$\pi_i'(e_i - q_i) - p < 0 \tag{30}$$

and similarly (23) with (25) reads

$$\pi_i'(e_i - q_i)(-1) + p - \lambda_i \leq 0. \tag{31}$$

If $\lambda_i = 0$, (30) and (31) yield the contradiction $0 < 0$. Should $\lambda_i > 0$, then $q_i = e_i$ and (31) reads $\pi_i'(0)(-1) + p - \lambda_i = 0$, which cannot happen as we have assumed that $\pi_i'(y_i) \to \infty$ as $y_i \to 0$, and the first claim is proved.

Turning to the second bullet point and considering a strategic *seller*, i.e. an agent i for whom $q_i > \frac{b_i}{p}$, yielding $\frac{B}{Q} > \frac{b_i}{q_i}$. Posit the converse of what is claimed, namely that

$$0 \leq \pi_i'(e_i - q_i + \frac{b_i}{p}) - p. \tag{32}$$

By using $\frac{\partial \Pi_i}{\partial b_i} \leq 0$ and rearranging (22), the right-hand side of (32) becomes

$$\leq \frac{1 - \frac{\partial p}{\partial b_i} q_i}{\frac{p - b_i \frac{\partial p}{\partial b_i}}{p^2}} - p = \frac{p^2(1 - \frac{1}{Q} q_i)}{p - b_i \frac{1}{Q}} - p. \tag{33}$$

Because $p = \frac{B}{Q}$, the right-hand side of the equality in (33)

$$= \frac{B}{Q}\left(\frac{\frac{B}{Q}\left(1 - \frac{q_i}{Q}\right)}{\frac{B}{Q} - \frac{b_i}{Q}} - 1\right) = \frac{B}{Q}\left(\frac{\frac{b_i}{Q} - \frac{B}{Q}\frac{q_i}{Q}}{\frac{B}{Q} - \frac{b_i}{Q}}\right) < \frac{B}{Q}\left(\frac{\frac{b_i}{Q} - \frac{b_i}{q_i}\frac{q_i}{Q}}{\frac{B}{Q} - \frac{b_i}{Q}}\right) = 0, \tag{34}$$

where the second equality is simply a consequence of a common denominator, whereas the last inequality follows by $\frac{B}{Q} > \frac{b_i}{q_i}$. Hence, (32), (33) and (34) say combined that $0 < 0$. A contradiction for a strategic buyer is obtained by the same arguments with reversing the inequalities in (32) and (34) and replacing the one in (33) with an equality, as a buyer must have $b_i > 0$; hence, $\frac{\partial \Pi_i}{\partial b_i} = 0$. □

Proof of Proposition 4.1. We start by spelling out the first- order conditions in the endowment withholding game with free supply, i.e. (9) with $\lambda_i = 0$, together with $\mu_i = 0$, by the proof of Proposition 3.3. With quadratic payoffs, the second market stage of the game yields

$$\alpha_i - \beta_i x_i = p \text{ with } p = \frac{\sum_{i \in I} \frac{\alpha_i}{\beta_i} - \sum_{i \in I} q_i}{\sum_{i \in I} \frac{1}{\beta_i}}, \tag{35}$$

and the formulas in (10) read $\frac{\partial p}{\partial q_i} = \frac{-1}{\sum_{j \in I} \frac{1}{\beta_j}}$ and $\frac{\partial x_i}{\partial q_i} = \frac{1}{\beta_i} \frac{1}{\sum_{j \in I} \frac{1}{\beta_j}}$. Thus, the first-order condition with respect to supply q_i as given in the second line in (9), now says

$$\begin{aligned} 0 &= (\pi_i'(e_i - q_i + x_i) - p) \cdot \left(-1 + \frac{\partial x_i}{\partial q_i}\right) + \frac{\partial p}{\partial q_i} \cdot (q_i - x_i) \\ &= (\alpha_i - \beta_i (e_i - q_i + x_i) - p) \cdot \left(-1 + \frac{\frac{1}{\beta_i}}{\sum_{j \in I} \frac{1}{\beta_j}}\right) + \frac{-1}{\sum_{j \in I} \frac{1}{\beta_j}} \cdot (q_i - x_i). \end{aligned} \tag{36}$$

We turn next to the technology manipulation game and start with the price that the stated profile $a_i = \alpha_i + \beta_i (e_i - q_i)$ for all $i \in I$ leads to. From (16) we get

$$r = \frac{\sum_{i\in I} \frac{\alpha_i+\beta_i(e_i-q_i)}{\beta_i} - \sum_{i\in I} e_i}{\sum_{i\in I} \frac{1}{\beta_i}} = \frac{\sum_{i\in I} \frac{\alpha_i}{\beta_i} - \sum_{i\in I} q_i}{\sum_{i\in I} \frac{1}{\beta_i}} = p$$

because of (35). Thus, $r = p$. Demand $y_i = (a_i - r)/\beta_i$, therefore reduces to $y_i = (\alpha_i + \beta_i (e_i - q_i) - p)/\beta_i = (e_i - q_i) + (\alpha_i - p)/\beta_i = e_i - q_i + x_i$, producing the same final allocation. With the assumed functional form, (17) yields

$$\frac{\partial r}{\partial a_i} = \frac{\frac{1}{\beta_i}}{\sum_{j\in I} \frac{1}{\beta_j}} \quad \text{and} \quad \frac{\partial y_i}{\partial a_i} = \frac{1}{\beta_i}\left(1 - \frac{\frac{1}{\beta_i}}{\sum_{j\in I} \frac{1}{\beta_j}}\right).$$

The left-hand side of the first-order condition (14) is therefore

$$\begin{aligned}
&= (\pi_i'(y_i) - r)\frac{\partial y_i}{\partial a_i} + \frac{\partial r}{\partial a_i}(e_i - y_i) \\
&= (\alpha_i - \beta_i (e_i - q_i + x_i) - p)\frac{\partial y_i}{\partial a_i} + \frac{\partial r}{\partial a_i}(e_i - (e_i - q_i + x_i)) \\
&= (\alpha_i - \beta_i (e_i - q_i + x_i) - p)\frac{1}{\beta_i}\left(1 - \frac{\frac{1}{\beta_i}}{\sum_{j\in I} \frac{1}{\beta_j}}\right) + \frac{\frac{1}{\beta_i}}{\sum_{j\in I} \frac{1}{\beta_j}}(q_i - x_i) \\
&= (\alpha_i - \beta_i (e_i - q_i + x_i) - p)\left(-1 + \frac{\frac{1}{\beta_i}}{\sum_{j\in I} \frac{1}{\beta_j}}\right) - \frac{1}{\sum_{j\in I} \frac{1}{\beta_j}}(q_i - x_i) \qquad (37)
\end{aligned}$$

The final equality is obtained by multiplying the previous one by $-\beta_i$. As (37) is nothing else than (36), the proof is complete. □

Proof of Proposition 4.2. The proof will be established as a contradiction to trade, after four lemmas.

Lemma A.1 (On vanishing margins). *Any feasible allocation implies that* $\pi_j'(e_j - q_j + \frac{b_j}{p}) = 0$ *for at least one* $j \in I$.

Proof (Trivial, but stated for completeness). Suppose on the contrary that $\pi_i'(e_i - q_i + \frac{b_i}{p}) > 0$ for all $i \in I$. For the latter to be true, $e_i - q_i + \frac{b_i}{p} < \hat{x}_i$ must hold for all $i \in I$. Final consumption summed over all agents equals

$$\sum_{i\in I}(e_i - q_i + \frac{b_i}{p}) = \sum_{i\in I} e_i < \sum_{i\in I} \hat{x}_i.$$

The equality follows by (20), whereas the inequality contradicts the fourth bullet point in Assumption 4.2 on aggregate hot air. Thus, there exists at least one agent

$j \in I$ with $\pi'_j(e_j - q_j + \frac{b_j}{p}) = 0$. $\square$

Lemma A.2 (On bids and supply when the margin is nil). *Suppose there exists an equilibrium where at least two agents have offered strictly positive supplies. Then, any agent j who has hot air in equilibrium, i.e. $\pi'_j(e_j - q_j + \frac{b_j}{p}) = 0$, will bid nothing and must have supplied $q_j \geq e_j - \hat{x}_j$.*

Proof. (22) for agent j amounts to

$$\pi'_j(e_j - q_j + \frac{b_j}{p}) \cdot \frac{p - b_j \frac{\partial p}{\partial b_j}}{p^2} + \frac{\partial p}{\partial b_j} q_j - 1 = \frac{1}{Q} q_j - 1 < 0$$

as $\pi'_j(\cdot) = 0$ and $q_j < Q$. Hence, for condition (24) to hold for agent j, we must have $b_j = 0$. Concerning his supply, if $e_j \leq \hat{x}_j$, and because $q_j \geq 0$, statement $q_j \geq e_j - \hat{x}_j$ follows trivially. Should $e_j > \hat{x}_j$ and $q_j < e_j - \hat{x}_j$, then by the definition of $\hat{x}_j$ and by the second bullet point in Assumption 4.2, it must be true that $\pi'_j(\cdot) = 0$ in equilibrium, so that the right-hand side of (23) reduces to

$$p + \frac{\partial p}{\partial q_j} q_j - \lambda_j = p \cdot \left(1 - \frac{q_j}{Q}\right) - \lambda_j. \tag{38}$$

Because $\hat{x}_j$ by definition is strictly positive, the constraint $q_j \leq e_j$ cannot bite when $q_j < e_j - \hat{x}_j$, hence λ_j will be nil. Because by assumption $q_j < Q$, expression (38) becomes strictly positive, contradicting condition (25). Thus, $q_j < e_j - \hat{x}_j$ is a contradiction yielding $q_j \geq e_j - \hat{x}_j$. $\square$

Lemma A.3 (On agents who bid). *Suppose there exists an equilibrium allocation where $\pi'_j(e_j - q_j + \frac{b_j}{p}) = 0$ for at least one $j \in I$. Then the total final consumption among $i \in I \backslash \{j\}$ is strictly greater than their aggregate business-as-usual emissions.*

Proof. Because, by assumption, there is one agent j with a vanishing margin, then by Lemma A.2, $b_j = 0$ and

$$p = \frac{\sum_{i \in I \backslash \{j\}} b_i}{\sum_{i \in I \backslash \{j\}} q_i + q_j}.$$

Suppose now, and contrary to what is claimed, that

$$\begin{aligned}
0 &\leq \sum_{i\in I\setminus\{j\}} \hat{x}_i - \sum_{i\in I\setminus\{j\}} (e_i - q_i + \frac{b_i}{p}) \\
&= \sum_{i\in I} \hat{x}_i - \hat{x}_j - \sum_{i\in I\setminus\{j\}} e_i + \sum_{i\in I\setminus\{j\}} q_i - \frac{\sum_{i\in I\setminus\{j\}} q_i + q_j}{\sum_{i\in I\setminus\{j\}} b_i} \sum_{i\in I\setminus\{j\}} b_i \\
&< \sum_{i\in I} e_i - \hat{x}_j - \sum_{i\in I\setminus\{j\}} e_i + \sum_{i\in I\setminus\{j\}} q_i - \sum_{i\in I} q_i \qquad (39) \\
&= \sum_{i\in I\setminus\{j\}} e_i + e_j - \hat{x}_j - \sum_{i\in I\setminus\{j\}} e_i - q_j \\
&= e_j - \hat{x}_j - q_j \leq 0,
\end{aligned}$$

a contradiction. The inequality in line (39) follows the fourth bullet point in Assumption 4.2, whereas the last inequality comes from Lemma A.2. □

Lemma A.4 (On the number of suppliers). *If an equilibrium with trade exists, then there are at least two agents with strictly positive supplies.*

Proof. Because there is trade, $Q > 0$. Suppose, on the contrary, that $q_i = Q$ for exactly one $i \in I$ with $q_j = 0$ for all else. By the third bullet point in Assumption 4.2, an agent $k \neq i$ with hot air must then exist, i.e. $e_k - \hat{x}_k > 0$. Because $q_k = 0$, agent k will in equilibrium have $\pi'_k(\cdot) = 0$. This implies that the right-hand side of (23) for agent k reduces to $p \cdot \left(1 - \frac{q_k}{Q}\right) - \lambda_k = p - \lambda_k = p > 0$ because $\lambda_k = 0$ (by the same argument as in Lemma A.2). The last inequality contradicts (25) for agent k. □

Proof of Proposition 4.2. Suppose that a Nash equilibrium with trade exists, i.e. B, Q and p are all > 0. By Lemma A.1, there will exist at least one agent with $\pi'_j(e_j - q_j + \frac{b_j}{p}) = 0$. From Lemma A.4, there must be more than one supplier. Thus, Lemma A.2 comes into effect and an agent with $\pi'_j(\cdot) = 0$ must have offered a bid $b_j = 0$ and supplied $q_j \geq e_j - \hat{x}_j$. By Lemma A.3, there will be hot air in aggregate among all $i \in I \setminus \{j\}$, and by Lemmas A.1 and A.2, there will once again exist at least one agent $i \in I \setminus \{j\}$, with $\pi'_i(e_i - q_i + \frac{b_i}{p}) = 0$, $b_i = 0$ and $q_i \geq e_i - \hat{x}_i$; and so on. By the logic of induction, this implies that all agents will choose $b_i = 0$, which contradicts $B > 0$. □

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